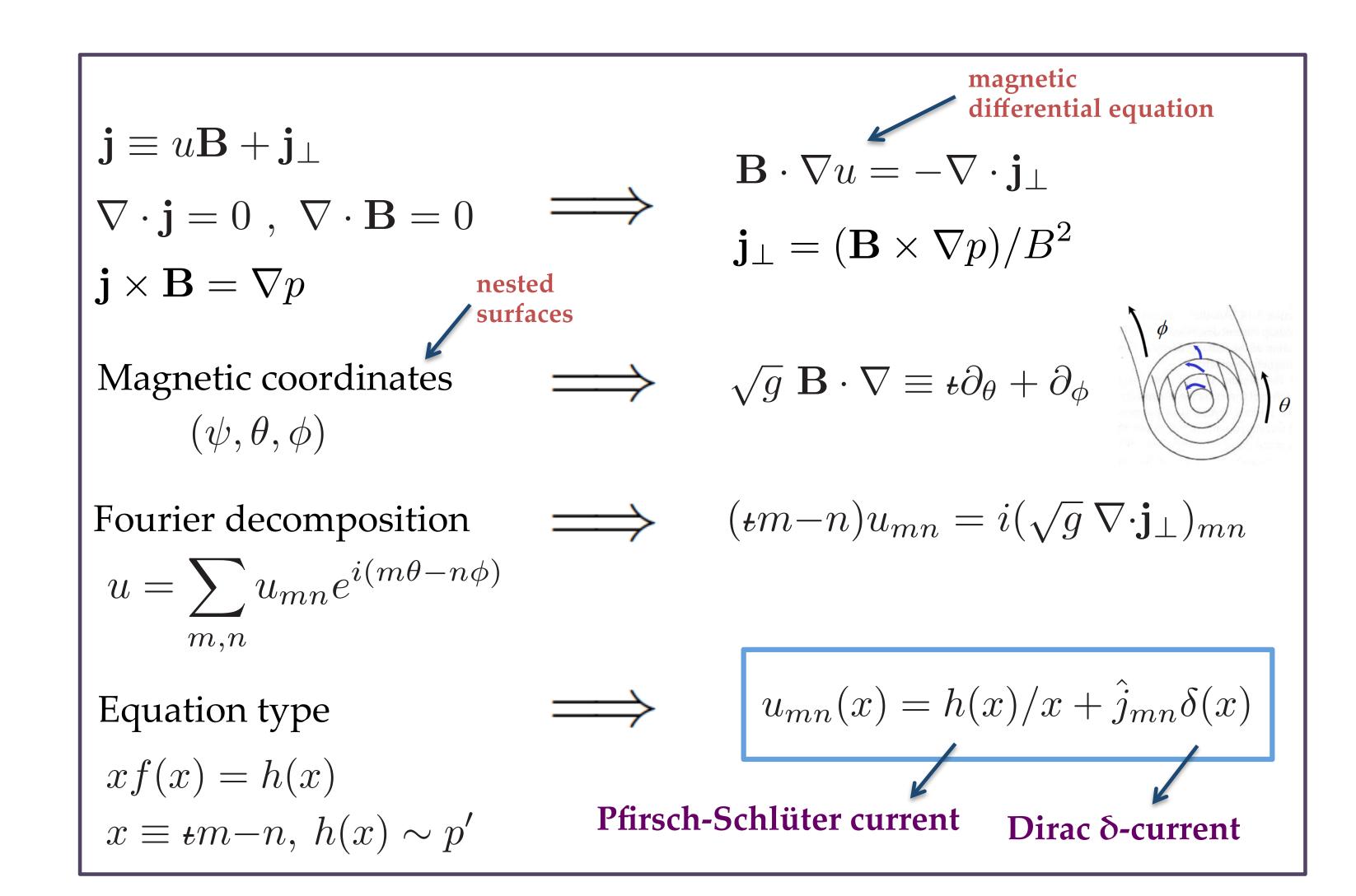
Plasma ideal response to a non-axisymmetric boundary perturbation



1. Motivation and Summary

- ► Main goal of ideal MHD: discovery of stable, magnetically confined plasma configurations [1].
- ▶ 2D MHD equilibrium problem is well posed (Grad-Shafranov equation).
- ▶ 3D MHD equilibrium problem is still today an outstanding issue.
- ▶ Ideal MHD predicts singular currents forming at rational surfaces in 3D equilibria [2]:
- \blacktriangleright A Pfirsch-Schlüter 1/x-current, which arises as a result of finite pressure gradient.
- A $\delta(x)$ -current which is necessary to prevent the formation of islands.
- Singular currents are critical for the ideal and resistive stability of 3D MHD equilibria [3-5].



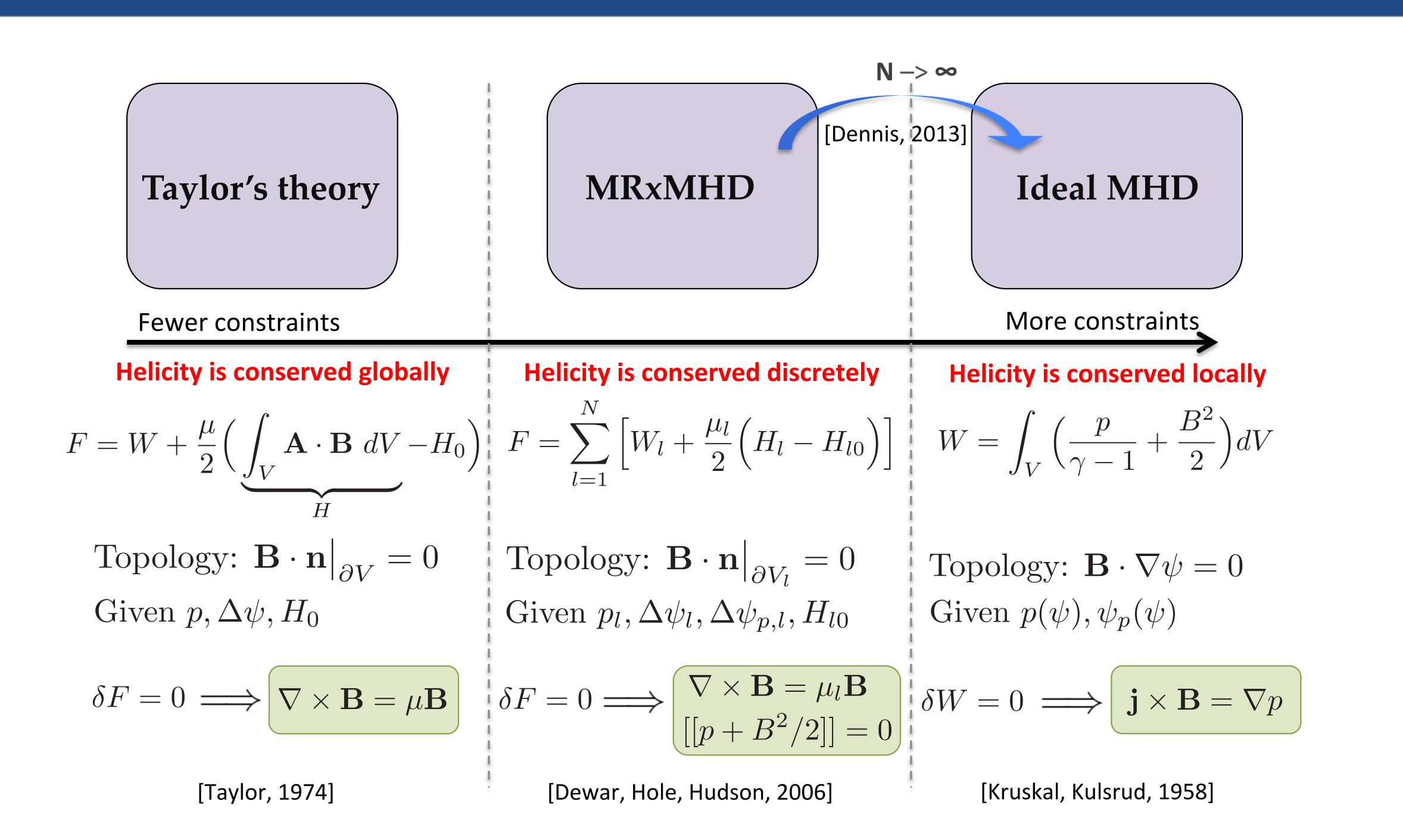
- ► A physically valid equilibrium must have finite integrated current densities:
- ullet δ -current densities are always integrable.
- ▶ 1/x pressure-driven current densities give divergent currents.
- ▶ Historical conclusion: 3D equilibria have either fractal [6] or stepped [7] pressure profiles.
- ▶ QUESTION: are there 3D MHD equilibria with nested surfaces and smooth pressure profiles?
- ► ANSWER: we present a new class of 3D MHD equilibria with
- (1) nested surfaces, arbitrary 3D geometry, and arbitrary smooth pressure
- (2) agreement between linear and nonlinear equilibrium calculations in the appropriate limit,
- (3) new predictions about the penetration of resonant boundary perturbations

J. Loizu^{1,2}, S. Hudson², A. Bhattacharjee², S. Lazerson² and P. Helander¹

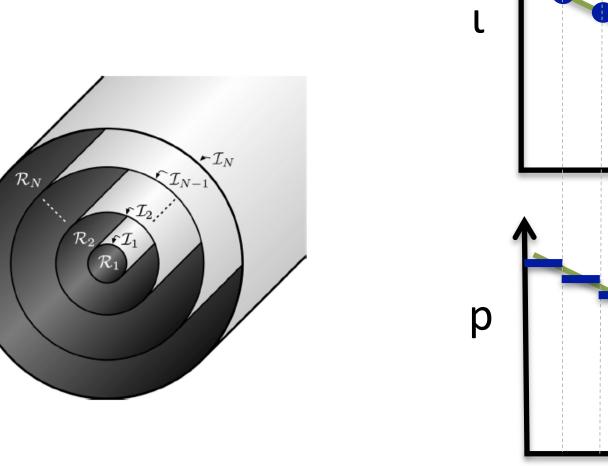
¹ Max-Planck-Institut für Plasmaphysik, Euratom Association, D-17491 Greifswald, Germany ² Princeton Plasma Physics Laboratory, Princeton New Jersey, USA

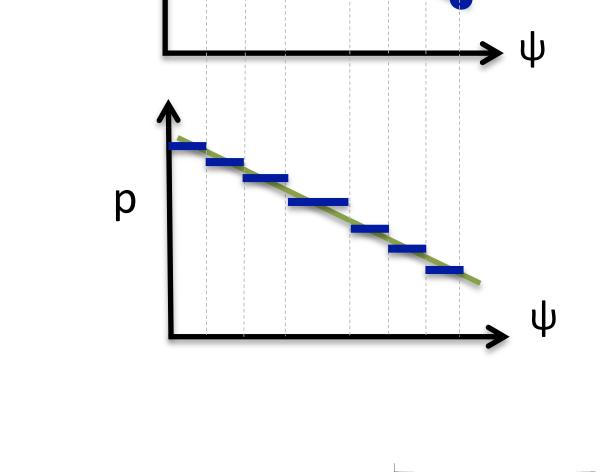
O) PPL

2. Exact computation of singular currents

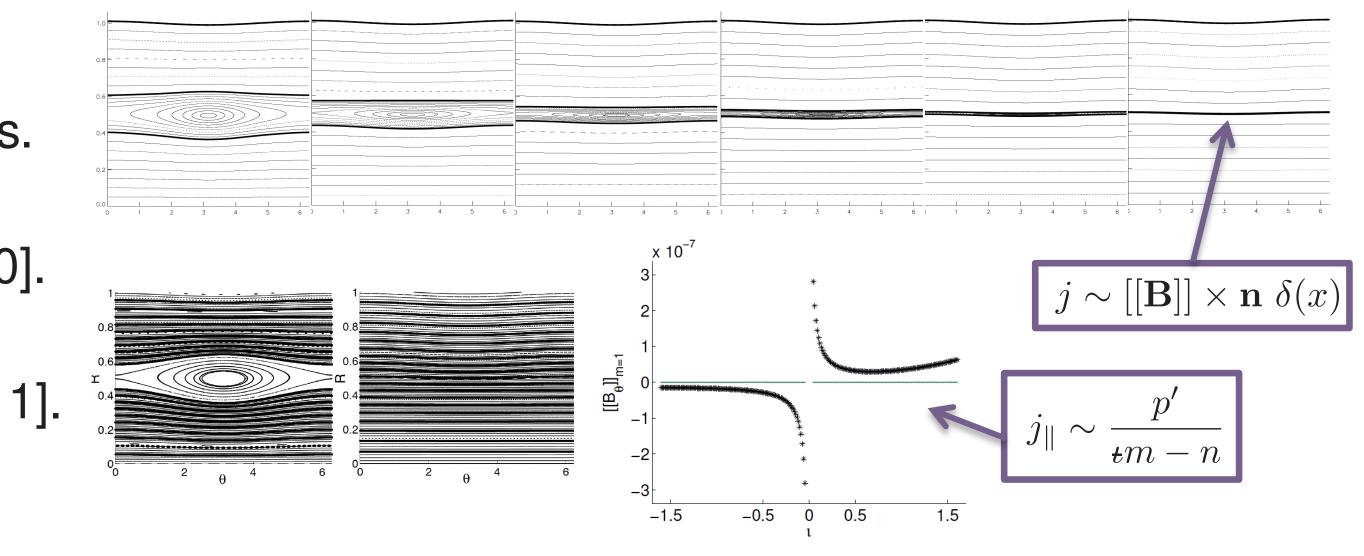


- ► Multiregion Relaxed MHD bridges Taylor's theory with ideal MHD.
- ► MRxMHD converges to ideal MHD for $N \to \infty$ [8].
- ► Stepped-pressure & singular currents are possible.
- ▶ The SPEC code [9] is a numerical implementation of MRxMHD.





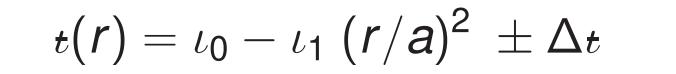
- ► SPEC used in **slab torus** to study singular currents.
- First numerical proof of their mutual existence [10].
- ► Exact verification against Hahm-Kulsrud theory [11].



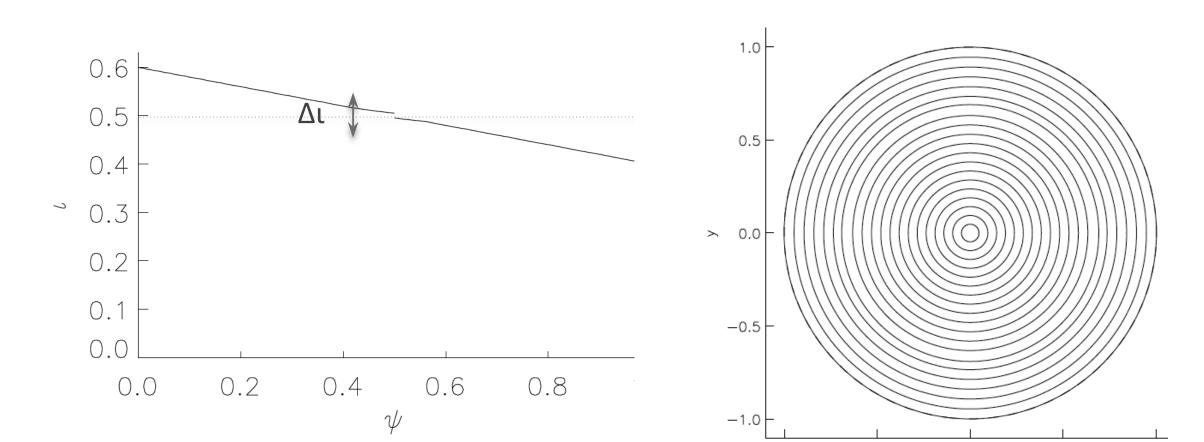
- ► Conclusion: locally infinite shear at the rational surfaces is required in order to have well-defined solutions.
- ► Corollary: consider 3D MHD equilibria with discontinuous transform (current sheet) across resonant surfaces. joaquim.loizu@ipp.mpg.de EPS conference, Lisbon, Portugal, June 2015

4. Cylindrical 3D MHD equilibria with current sheets

Screw pinch axisymmetric equilibrium with $p(r) \equiv 0$ and



where Δ_t manifests in the form of a "DC" current sheet.



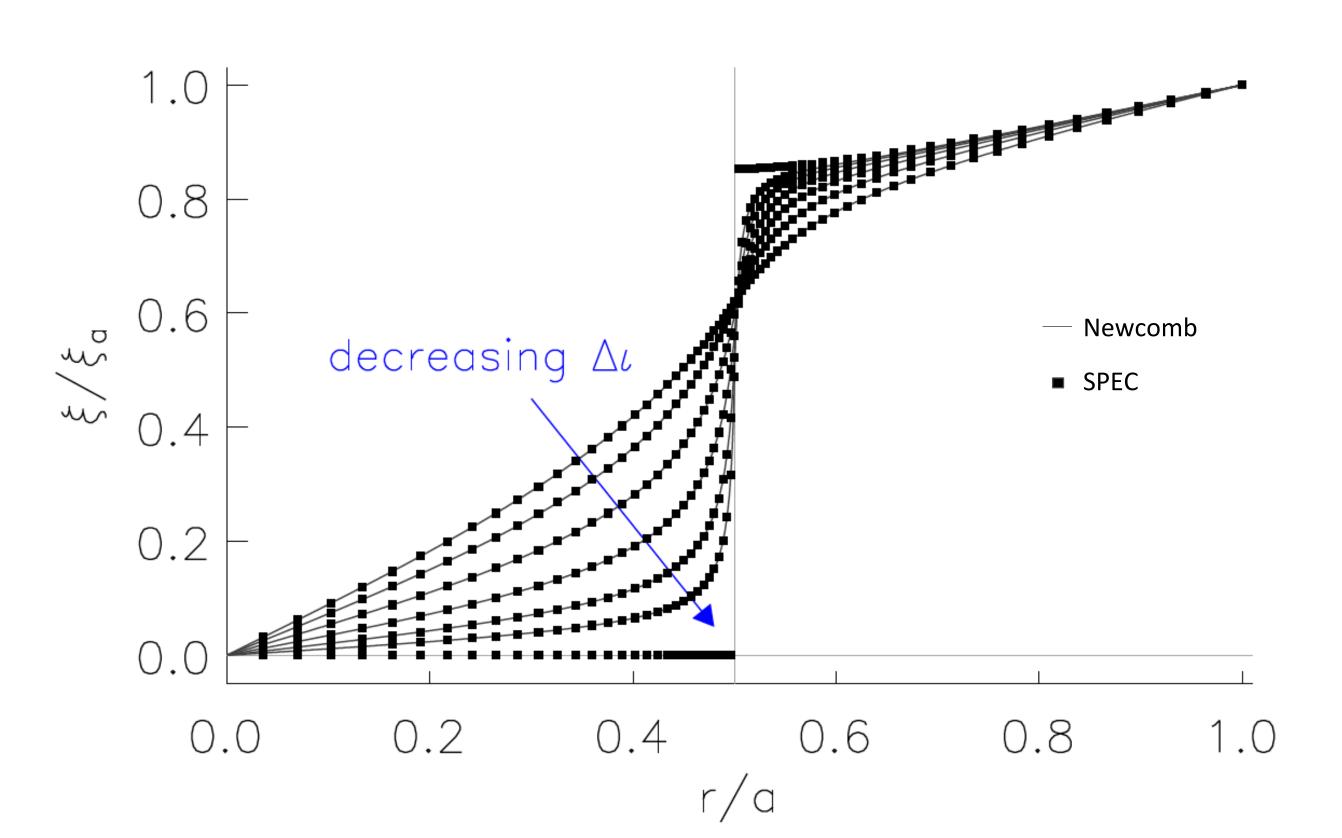
Linear response to boundary perturbation $\xi_a \cos(m\theta + kz)$:

$$rac{d}{dr}\left(frac{d\xi}{dr}
ight)-g\xi=0$$
 (Newcomb's equation)

where f(r) and g(r) depend on the equilibrium.

▶ Radial displacement, $\xi(r)$, gives overlap of surfaces unless

$$|rac{d\xi}{dr}|<1$$
 (sine qua non condition)



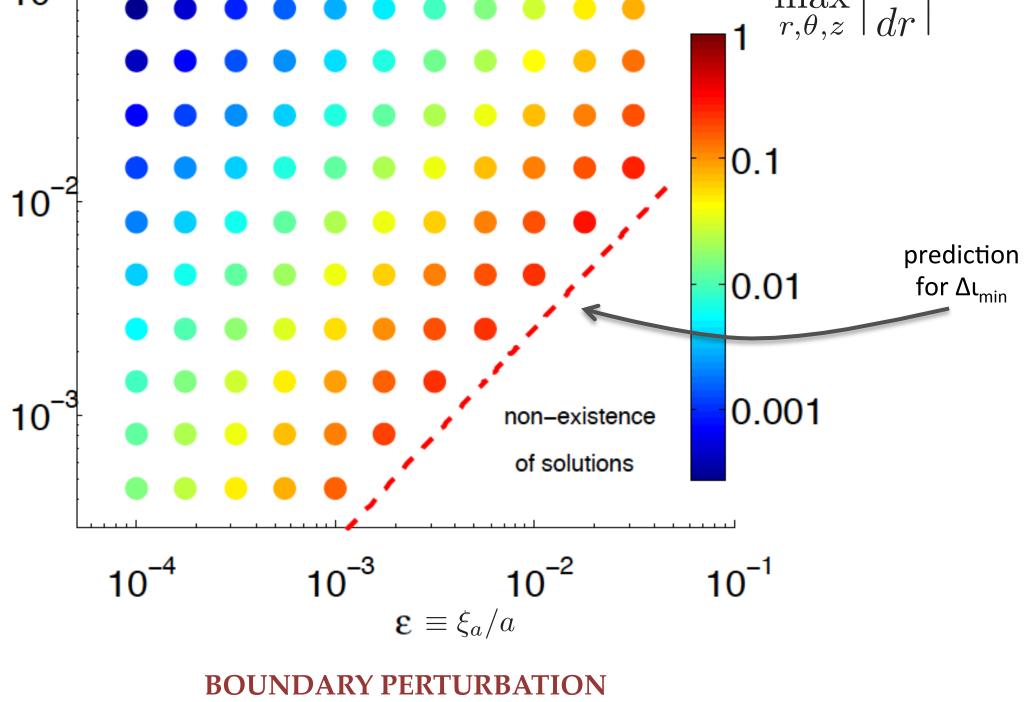
► Analytical expression for $|d\xi/dr|_{r=r_s}$

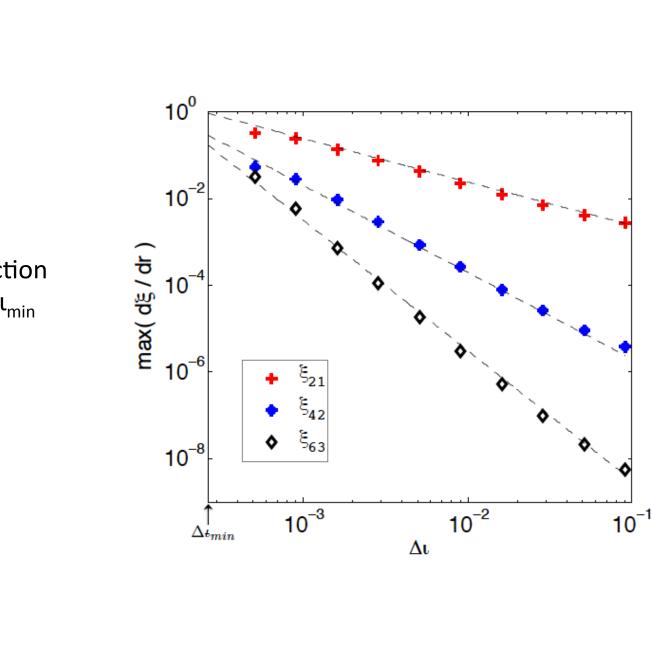
$$\xi_{\mathcal{S}}' = 2 t_{\mathcal{S}}' rac{\xi_{\mathcal{S}}}{\Delta_t}$$

provides a minimum current sheet

$$\Delta t > \Delta t_{ extit{min}} = 2 t_{ extit{s}}' \xi_{ extit{s}} pprox t_{ extit{s}}' \xi_{ extit{a}}$$

as the *sine qua non* condition for the existence of equilibria.





- Confirmed by nonlinear simulations.
- ► Conclusions: (1) Boundary perturbation penetrates all the way into the centre of a tokamak, even within ideal MHD. (2) 3D ideal equilibria exist as long as $\Delta t > \Delta t_{min}$, and may be computed with arbitrary smooth pressure.

7. References

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